

# The Thirty-Third Annual Eastern Shore High School Mathematics Competition

November 10, 2016

## Individual Contest Exam

### Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

**There is no penalty for incorrect answers.**

The use of a calculator is **not** permitted on this exam.

In the event of tie scores, #18, #19 and #20 will be used as tiebreakers.

1. If  $\log_{10} 3x = \frac{1}{4} \log_{10} (x - 12)^4$ , then
- a.  $x = 6$       b.  $x = 3$       c.  $x = 6$       d.  $x = 3$  or  $x = 6$       e. the equation has no real solutions.
2. The sum of the integer-valued solutions of  $j^3 - 2x = x^2j - 3 - 2x - x^2$  is
- a. -5      b. -2      c. -3      d. 2      e. 5
3. Late one Saturday night, Anna, Bill and CeCe are having a grand old time doing arithmetic with the prime numbers. Anna finds the product of the three smallest primes and calls her product  $A$ . Bill finds the sum of the next five primes and calls his sum  $B$ . CeCe then computes  $C = A - B$ . What is the value of  $C$ ?
- a. 77      b. 97      c. 2010      d. 3,233,230      e. 9,699,690
4. If  $\frac{4}{w} + \frac{4}{x} = \frac{4}{y}$  and  $wx = y$ , then the average (arithmetic mean) of  $w$  and  $x$  is
- a.  $\frac{1}{2}$       b. 1      c. 2      d. 4      e. 8
5. Infinitely many circles are constructed so that the radius of the first is 1, the radius of the second is  $\frac{1}{2}$ , the radius of the third is  $\frac{1}{4}$ , and so on (that is, starting with a circle of radius of 1, each subsequent circles radius is half the radius of the previous circle). What is the sum of the areas of these infinitely many circles?
- a.  $\frac{2}{2}$       b.  $\frac{7}{8}$       c.  $\frac{8}{7}$       d.  $\frac{4}{3}$       e. 2
6. Consider an exponential function of the form  $f(x) = ae^{bx}$ , where  $a$  and  $b$  are constants. If  $f(2) = e^{-2}$  and  $f(3) = e^{-4}$ , what is  $f(1)$ ?
- a.  $e^{-4}$       b. 1      c. 2      d.  $e^2$       e.  $e^4$
7. A class consists of ten students. The teacher failed to record one student's test grade. The sum of the grades that she had recorded was 698, and the moided toe

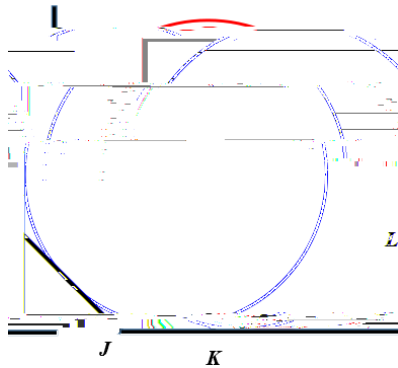
9. Statistics for grades on math tests from three different classes are provided in the table below.

|         | mean | standard deviation |
|---------|------|--------------------|
| Class 1 | 79.1 | 5.7                |
| Class 2 | 80.6 | 8.4                |
| Class 3 | 75.9 | 15.3               |

Bobbi is in Class 1, Mary is in Class 2, and Jenny is in Class 3. All three students earned a grade of 81. Who did the best relative to her classmates?

- There is no difference among the three students.
- Bobbi did the best relative to her classmates.
- Mary did the best relative to her classmates.
- Jenny did the best relative to her classmates.
- Additional information about each class is needed.

10. In the figure below  $\overline{JK} \perp \overline{JL}$  and the circle is tangent to  $\overline{JK}$ ,  $\overline{JL}$ , and  $\overline{KL}$ . If  $d$  is the length of the diameter of the circle, then



- $d = (JK)^2$
- $d = (JL)^2$
- $d = (JK)^2 + (JL)^2$
- $d = (JK + KL + JL)$
- cannot be determined

11. How many distinct factors does 2016 have?

- 8
- 12
- 16
- 24
- 36

12. Which one of the following series has the value of 2016?

- $\sum_{n=0}^{\infty} 2016 \left(\frac{1}{2}\right)^n$
- $\sum_{n=0}^{\infty} 1512 \left(\frac{1}{4}\right)^n$
- $\sum_{n=0}^{\infty} 1008 \left(\frac{1}{8}\right)^n$
- $\sum_{n=0}^{\infty} 1764 \left(\frac{1}{16}\right)^n$
- $\sum_{n=0}^{\infty} 1890 \left(\frac{1}{32}\right)^n$

13. If  $\log_2(x) = c$ , then  $\log_8(x) + \log_4(x) - \log_{1/2}(x)$  equals

- a.  $\frac{11}{6}c$       b.  $6c$       c.  $c^3 + c^2 + c^{-1}$       d.  $x^3 + x^2 + x^{-1}$       e.  $6x$

14. Each of the 75 people in a room is wearing at least one colored bracelet. The numbers of red bracelets, yellow bracelets,

19. In the diagram, arc  $AC$  is part of a circle, and the segment  $BC$  is tangent to this circle. The angle  $ABC$  is a right angle. Based on the measurements given, the radius of this circle



- a. is 13      b. is  $\frac{169}{10}$       c. is 17      d. is 12      e. cannot be determined.
20. Consider the sequence shown below

7; 9; 12; 13; 15; 18; 19; 21; 24; 25; 27; 30; 31; 33; 36; 37; ...

What is the sum of the first 99 terms of the sequence?

- a. 6436      b. 8218      c. 9306      d. 10395      e. 10428